

- Output voor probleem 9 uit Pelletier's lijst.

===== Proof: =====

With () and AXIOM : 10. p,p	- p,p
With (10) and Left- ~ : 9. ~p,p,p	- p
With () and AXIOM : 11. p,p,q	- p
With (9,11) and Left- V : 8. (~p V q),p,p	- p
With () and AXIOM : 14. p,q	- p,p
With (14) and Left- ~ : 13. ~p,p,q	- p
With () and AXIOM : 15. p,q,q	- p
With (13,15) and Left- V : 12. (~p V q),p,q	- p
With (8,12) and Left- V : 7. (p V q),(~p V q),p	- p
With (7) and Left- ~ : 6. ~p,(p V q),(~p V q),p	-
With () and AXIOM : 21. p	- p,q,p
With (21) and Left- ~ : 20. ~p,p	- p,q
With (20) and Left- ~ : 19. ~q,p,~p	- p
With () and AXIOM : 23. p,q	- p,q
With (23) and Left- ~ : 22. ~q,p,q	- p
With (19,22) and Left- V : 18. (~p V q),~q,p	- p
With () and AXIOM : 27. q	- p,q,p
With (27) and Left- ~ : 26. ~p,q	- p,q
With (26) and Left- ~ : 25. ~q,q,~p	- p
With () and AXIOM : 29. q,q	- p,q
With (29) and Left- ~ : 28. ~q,q,q	- p
With (25,28) and Left- V : 24. (~p V q),~q,q	- p
With (18,24) and Left- V : 17. (p V q),(~p V q),~q	- p
With (17) and Left- ~ : 16. ~p,(p V q),(~p V q),~q	-
With (6,16) and Left- V : 5. (p V ~q),~p,(p V q),(~p V q)	-
With (5) and Left- ^ : 4. ((p V q) ^ (~p V q)),(p V ~q),~p	-
With () and AXIOM : 36. p,p	- q,p
With (36) and Left- ~ : 35. ~p,p,p	- q
With () and AXIOM : 37. p,p,q	- q
With (35,37) and Left- V : 34. (~p V q),p,p	- q
With () and AXIOM : 40. p,q	- q,p
With (40) and Left- ~ : 39. ~p,p,q	- q
With () and AXIOM : 41. p,q,q	- q
With (39,41) and Left- V : 38. (~p V q),p,q	- q
With (34,38) and Left- V : 33. (p V q),(~p V q),p	- q
With (33) and Left- ~ : 32. ~q,(p V q),(~p V q),p	-
With () and AXIOM : 47. p	- q,q,p
With (47) and Left- ~ : 46. ~p,p	- q,q
With (46) and Left- ~ : 45. ~q,p,~p	- q
With () and AXIOM : 49. p,q	- q,q
With (49) and Left- ~ : 48. ~q,p,q	- q
With (45,48) and Left- V : 44. (~p V q),~q,p	- q
With () and AXIOM : 53. q	- q,q,p
With (53) and Left- ~ : 52. ~p,q	- q,q
With (52) and Left- ~ : 51. ~q,q,~p	- q
With () and AXIOM : 55. q,q	- q,q
With (55) and Left- ~ : 54. ~q,q,q	- q
With (51,54) and Left- V : 50. (~p V q),~q,q	- q
With (44,50) and Left- V : 43. (p V q),(~p V q),~q	- q
With (43) and Left- ~ : 42. ~q,(p V q),(~p V q),~q	-
With (32,42) and Left- V : 31. (p V ~q),~q,(p V q),(~p V q)	-

With (31) and Left-  $\wedge$  : 30.  $((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q), \sim q \quad \vdash$   
 With (4,30) and Left-  $\vee$  : 3.  $(\sim p \vee \sim q), ((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q) \quad \vdash$   
 With (3) and Left-  $\wedge$  : 2.  $((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q), (\sim p \vee \sim q) \quad \vdash$   
 With (2) and Right-  $\sim$  : 1.  $((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q) \quad \vdash \sim(\sim p \vee \sim q)$   
 With (1) and Right-  $\Rightarrow$  : 0.  $\vdash (((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q))$   
 $\Rightarrow \sim(\sim p \vee \sim q)$

===== QED ==

- Listing van een complete trace voor probleem 9 uit Pelletier's lijst.1

Adding initial proposition to RHS... (Trying to find countermodel)

Fireing up ATP:

Performing iteration with:

LHS:

RHS:  $((((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q)) \Rightarrow \sim(\sim p \vee \sim q))$

Non-atomic proposition found on RHS...

Iterating on the right-hand side...

0. Right-  $\Rightarrow$  on  $((((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q)) \Rightarrow \sim(\sim p \vee \sim q))$  with

LHS:

RHS:

Performing iteration with:

LHS:  $((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q)$

RHS:  $\sim(\sim p \vee \sim q)$

Non-atomic proposition found on RHS...

Iterating on the right-hand side...

1. Right-  $\sim$  on  $\sim(\sim p \vee \sim q)$  with

LHS:  $((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q)$

RHS:

Performing iteration with:

LHS:  $((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q), (\sim p \vee \sim q)$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

2. Left-  $\wedge$  on  $((p \vee q) \wedge (\sim p \vee q)) \wedge (p \vee \sim q)$  with

LHS:  $\sim p \vee \sim q$

RHS:

Performing iteration with:

LHS:  $\sim p \vee \sim q, ((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q)$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

3. Left-  $\vee$  on  $\sim p \vee \sim q$  with

LHS:  $((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q)$

RHS:

left split:

Performing iteration with:

LHS:  $((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q), \sim p$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

4. Left-  $\wedge$  on  $((p \vee q) \wedge (\sim p \vee q))$  with

LHS:  $(p \vee \sim q), \sim p$

RHS:

Performing iteration with:

LHS:  $(p \vee \sim q), \sim p, (p \vee q), (\sim p \vee q)$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

5. Left-  $\vee$  on  $(p \vee \sim q)$  with

LHS:  $\sim p, (p \vee q), (\sim p \vee q)$

RHS:

left split:

Performing iteration with:

LHS:  $\sim p, (p \vee q), (\sim p \vee q), p$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

6. Left-  $\sim$  on  $\sim p$  with

LHS:  $(p \vee q), (\sim p \vee q), p$

RHS:

Performing iteration with:

LHS:  $(p \vee q), (\sim p \vee q), p$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

7. Left-  $\vee$  on  $(p \vee q)$  with

LHS:  $(\sim p \vee q), p$

RHS:  $p$

left split:

Performing iteration with:

LHS:  $(\sim p \vee q), p, p$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

8. Left-  $\vee$  on  $(\sim p \vee q)$  with

LHS:  $p, p$

RHS:  $p$

left split:

Performing iteration with:

LHS:  $p, p, \sim p$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

9. Left-  $\sim$  on  $\sim p$  with

LHS:  $p, p$

RHS:  $p$

Performing iteration with:

LHS:  $p, p$

RHS:  $p, p$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

$[p, p]$

$[p, p]$

Checking if  $p$  is contained in

$[p, p]$

True

Sequent is Axiom!

No contradiction.

Close branch.

9. yields:

LHS:  $[p, p]$

RHS:  $[p, p]$

8. yields:

LHS:  $[p, p, \sim p]$   
RHS:  $[p]$

right split:

Performing iteration with:

LHS:  $p, p, q$

RHS:  $p$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

$[p, p, q]$

$[p]$

Checking if  $p$  is contained in

$[p]$

True

Sequent is Axiom!

No contradiction.

Close branch.

8. yields:

LHS:  $[p, p, q]$

RHS:  $[p]$

7. yields:

LHS:  $[(\sim p \vee q), p, p]$

RHS:  $[p]$

right split:

Performing iteration with:

LHS:  $(\sim p \vee q), p, q$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

12. Left-  $\vee$  on  $(\sim p \vee q)$  with

LHS:  $p, q$

RHS:  $p$

left split:

Performing iteration with:

LHS:  $p, q, \sim p$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

13. Left-  $\sim$  on  $\sim p$  with

LHS:  $p, q$

RHS:  $p$

Performing iteration with:

LHS:  $p, q$

RHS:  $p, p$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

$[p, q]$

$[p, p]$

Checking if  $p$  is contained in

$[p, p]$

True

Sequent is Axiom!

No contradiction.

Close branch.

13. yields:

LHS:  $[p, q]$

RHS:  $[p,p]$

12. yields:

LHS:  $[p,q,\sim p]$

RHS:  $[p]$

right split:

Performing iteration with:

LHS:  $p,q,q$

RHS:  $p$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

$[p,q,q]$

$[p]$

Checking if  $p$  is contained in

$[p]$

True

Sequent is Axiom!

No contradiction.

Close branch.

12. yields:

LHS:  $[p,q,q]$

RHS:  $[p]$

7. yields:

LHS:  $[(\sim p \vee q),p,q]$

RHS:  $[p]$

6. yields:

LHS:  $[(p \vee q),(\sim p \vee q),p]$

RHS:  $[p]$

5. yields:

LHS:  $[\sim p,(p \vee q),(\sim p \vee q),p]$

RHS:  $[\ ]$

right split:

Performing iteration with:

LHS:  $\sim p,(p \vee q),(\sim p \vee q),\sim q$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

16. Left-  $\sim$  on  $\sim p$  with

LHS:  $(p \vee q),(\sim p \vee q),\sim q$

RHS:

Performing iteration with:

LHS:  $(p \vee q),(\sim p \vee q),\sim q$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

17. Left-  $\vee$  on  $(p \vee q)$  with

LHS:  $(\sim p \vee q),\sim q$

RHS:  $p$

left split:

Performing iteration with:

LHS:  $(\sim p \vee q),\sim q,p$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

18. Left-  $\vee$  on  $(\sim p \vee q)$  with

LHS:  $\sim q, p$

RHS:  $p$

left split:

Performing iteration with:

LHS:  $\sim q, p, \sim p$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

19. Left-  $\sim$  on  $\sim q$  with

LHS:  $p, \sim p$

RHS:  $p$

Performing iteration with:

LHS:  $p, \sim p$

RHS:  $p, q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

20. Left-  $\sim$  on  $\sim p$  with

LHS:  $p$

RHS:  $p, q$

Performing iteration with:

LHS:  $p$

RHS:  $p, q, p$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

[ $p$ ]

[ $p, q, p$ ]

Checking if  $p$  is contained in

[ $p, q, p$ ]

True

Sequent is Axiom!

No contradiction.

Close branch.

20. yields:

LHS: [ $p$ ]

RHS: [ $p, q, p$ ]

19. yields:

LHS: [ $p, \sim p$ ]

RHS: [ $p, q$ ]

18. yields:

LHS: [ $\sim q, p, \sim p$ ]

RHS: [ $p$ ]

right split:

Performing iteration with:

LHS:  $\sim q, p, q$

RHS:  $p$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

22. Left-  $\sim$  on  $\sim q$  with

LHS:  $p, q$

RHS:  $p$

Performing iteration with:

LHS:  $p, q$

RHS:  $p, q$

LHS & RHS contain only atoms!  
Checking if an atom in LHS also resides in RHS:

[p,q]

[p,q]

Checking if p is contained in

[p,q]

True

Sequent is Axiom!

No contradiction.

Close branch.

22. yields:

LHS: [p,q]

RHS: [p,q]

18. yields:

LHS: [ $\sim$ q,p,q]

RHS: [p]

17. yields:

LHS: [ $(\sim$ p  $\vee$  q), $\sim$ q,p]

RHS: [p]

right split:

Performing iteration with:

LHS: ( $\sim$ p  $\vee$  q), $\sim$ q,q

RHS: p

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

24. Left-  $\vee$  on ( $\sim$ p  $\vee$  q) with

LHS:  $\sim$ q,q

RHS: p

left split:

Performing iteration with:

LHS:  $\sim$ q,q, $\sim$ p

RHS: p

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

25. Left-  $\sim$  on  $\sim$ q with

LHS: q, $\sim$ p

RHS: p

Performing iteration with:

LHS: q, $\sim$ p

RHS: p,q

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

26. Left-  $\sim$  on  $\sim$ p with

LHS: q

RHS: p,q

Performing iteration with:

LHS: q

RHS: p,q,p

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

[q]

[p,q,p]

Checking if q is contained in

[p,q,p]

True

Sequent is Axiom!

No contradiction.

Close branch.

26. yields:

LHS: [q]

RHS: [p,q,p]

25. yields:

LHS: [q,~p]

RHS: [p,q]

24. yields:

LHS: [~q,q,~p]

RHS: [p]

right split:

Performing iteration with:

LHS: ~q,q,q

RHS: p

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

28. Left- ~ on ~q with

LHS: q,q

RHS: p

Performing iteration with:

LHS: q,q

RHS: p,q

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

[q,q]

[p,q]

Checking if q is contained in

[p,q]

True

Sequent is Axiom!

No contradiction.

Close branch.

28. yields:

LHS: [q,q]

RHS: [p,q]

24. yields:

LHS: [~q,q,q]

RHS: [p]

17. yields:

LHS: [(~p ∨ q),~q,q]

RHS: [p]

16. yields:

LHS: [(p ∨ q),(~p ∨ q),~q]

RHS: [p]

5. yields:

LHS: [~p,(p ∨ q),(~p ∨ q),~q]

RHS: []

4. yields:



LHS:  $[(p \vee \sim q), \sim p, (p \vee q), (\sim p \vee q)]$   
RHS:  $\square$

3. yields:

LHS:  $[((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q), \sim p]$   
RHS:  $\square$

right split:

Performing iteration with:

LHS:  $((p \vee q) \wedge (\sim p \vee q)), (p \vee \sim q), \sim q$   
RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

30. Left-  $\wedge$  on  $((p \vee q) \wedge (\sim p \vee q))$  with

LHS:  $(p \vee \sim q), \sim q$

RHS:

Performing iteration with:

LHS:  $(p \vee \sim q), \sim q, (p \vee q), (\sim p \vee q)$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

31. Left-  $\vee$  on  $(p \vee \sim q)$  with

LHS:  $\sim q, (p \vee q), (\sim p \vee q)$

RHS:

left split:

Performing iteration with:

LHS:  $\sim q, (p \vee q), (\sim p \vee q), p$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

32. Left-  $\sim$  on  $\sim q$  with

LHS:  $(p \vee q), (\sim p \vee q), p$

RHS:

Performing iteration with:

LHS:  $(p \vee q), (\sim p \vee q), p$

RHS:  $q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

33. Left-  $\vee$  on  $(p \vee q)$  with

LHS:  $(\sim p \vee q), p$

RHS:  $q$

left split:

Performing iteration with:

LHS:  $(\sim p \vee q), p, p$

RHS:  $q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

34. Left-  $\vee$  on  $(\sim p \vee q)$  with

LHS:  $p, p$

RHS:  $q$

left split:

Performing iteration with:

LHS:  $p, p, \sim p$

RHS:  $q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

35. Left-  $\sim$  on  $\sim p$  with

LHS:  $p, p$

RHS: q  
Performing iteration with:  
LHS: p,p  
RHS: q,p  
LHS & RHS contain only atoms!  
Checking if an atom in LHS also resides in RHS:  
[p,p]  
[q,p]  
Checking if p is contained in  
[q,p]  
True  
Sequent is Axiom!  
No contradiction.  
Close branch.  
35. yields:  
LHS: [p,p]  
RHS: [q,p]

34. yields:  
LHS: [p,p,~p]  
RHS: [q]

right split:  
Performing iteration with:  
LHS: p,p,q  
RHS: q  
LHS & RHS contain only atoms!  
Checking if an atom in LHS also resides in RHS:  
[p,p,q]  
[q]  
Checking if p is contained in  
[q]  
False  
Checking if p is contained in  
[q]  
False  
Checking if q is contained in  
[q]  
True  
Sequent is Axiom!  
No contradiction.  
Close branch.  
34. yields:  
LHS: [p,p,q]  
RHS: [q]

33. yields:  
LHS: [(~p ∨ q),p,p]  
RHS: [q]

right split:  
Performing iteration with:  
LHS: (~p ∨ q),p,q  
RHS: q  
Non-atomic proposition found on LHS...  
Iterating on the left-hand side...  
38. Left-∨ on (~p ∨ q) with  
LHS: p,q

RHS: q  
 left split:  
 Performing iteration with:  
 LHS: p,q,~p  
 RHS: q  
 Non-atomic proposition found on LHS...  
 Iterating on the left-hand side...  
 39. Left- ~ on ~p with  
 LHS: p,q  
 RHS: q  
 Performing iteration with:  
 LHS: p,q  
 RHS: q,p  
 LHS & RHS contain only atoms!  
 Checking if an atom in LHS also resides in RHS:  
 [p,q]  
 [q,p]  
 Checking if p is contained in  
 [q,p]  
 True  
 Sequent is Axiom!  
 No contradiction.  
 Close branch.  
 39. yields:  
 LHS: [p,q]  
 RHS: [q,p]

38. yields:  
 LHS: [p,q,~p]  
 RHS: [q]

right split:  
 Performing iteration with:  
 LHS: p,q,q  
 RHS: q  
 LHS & RHS contain only atoms!  
 Checking if an atom in LHS also resides in RHS:  
 [p,q,q]  
 [q]  
 Checking if p is contained in  
 [q]  
 False  
 Checking if q is contained in  
 [q]  
 True  
 Sequent is Axiom!  
 No contradiction.  
 Close branch.  
 38. yields:  
 LHS: [p,q,q]  
 RHS: [q]

33. yields:  
 LHS: [(~p ∨ q),p,q]  
 RHS: [q]

32. yields:  
 LHS: [(p ∨ q) (~p ∨ q) p]

RHS: [q]

31. yields:

LHS: [ $\sim q, (p \vee q), (\sim p \vee q), p$ ]

RHS: []

right split:

Performing iteration with:

LHS:  $\sim q, (p \vee q), (\sim p \vee q), \sim q$

RHS:

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

42. Left-  $\sim$  on  $\sim q$  with

LHS:  $(p \vee q), (\sim p \vee q), \sim q$

RHS:

Performing iteration with:

LHS:  $(p \vee q), (\sim p \vee q), \sim q$

RHS: q

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

43. Left-  $\vee$  on  $(p \vee q)$  with

LHS:  $(\sim p \vee q), \sim q$

RHS: q

left split:

Performing iteration with:

LHS:  $(\sim p \vee q), \sim q, p$

RHS: q

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

44. Left-  $\vee$  on  $(\sim p \vee q)$  with

LHS:  $\sim q, p$

RHS: q

left split:

Performing iteration with:

LHS:  $\sim q, p, \sim p$

RHS: q

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

45. Left-  $\sim$  on  $\sim q$  with

LHS:  $p, \sim p$

RHS: q

Performing iteration with:

LHS:  $p, \sim p$

RHS: q, q

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

46. Left-  $\sim$  on  $\sim p$  with

LHS: p

RHS: q, q

Performing iteration with:

LHS: p

RHS: q, q, p

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

[p]

[q, q, p]

Checking if p is contained in

[q, q, p]

True  
Sequent is Axiom!  
No contradiction.  
Close branch.

46. yields:  
LHS: [p]  
RHS: [q,q,p]

45. yields:  
LHS: [p,~p]  
RHS: [q,q]

44. yields:  
LHS: [~q,p,~p]  
RHS: [q]

right split:  
Performing iteration with:  
LHS: ~q,p,q  
RHS: q  
Non-atomic proposition found on LHS...  
Iterating on the left-hand side...  
48. Left- ~ on ~q with  
LHS: p,q  
RHS: q  
Performing iteration with:  
LHS: p,q  
RHS: q,q  
LHS & RHS contain only atoms!  
Checking if an atom in LHS also resides in RHS:  
[p,q]  
[q,q]  
Checking if p is contained in  
[q,q]  
False  
Checking if q is contained in  
[q,q]  
True  
Sequent is Axiom!  
No contradiction.  
Close branch.

48. yields:  
LHS: [p,q]  
RHS: [q,q]

44. yields:  
LHS: [~q,p,q]  
RHS: [q]

43. yields:  
LHS: [(~p ∨ q),~q,p]  
RHS: [q]

right split:  
Performing iteration with:  
LHS: (~p ∨ q),~q,q  
RHS: q  
Non-atomic proposition found on LHS

Iterating on the left-hand side...

50. Left-  $\vee$  on  $(\sim p \vee q)$  with

LHS:  $\sim q, q$

RHS:  $q$

left split:

Performing iteration with:

LHS:  $\sim q, q, \sim p$

RHS:  $q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

51. Left-  $\sim$  on  $\sim q$  with

LHS:  $q, \sim p$

RHS:  $q$

Performing iteration with:

LHS:  $q, \sim p$

RHS:  $q, q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

52. Left-  $\sim$  on  $\sim p$  with

LHS:  $q$

RHS:  $q, q$

Performing iteration with:

LHS:  $q$

RHS:  $q, q, p$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

[ $q$ ]

[ $q, q, p$ ]

Checking if  $q$  is contained in

[ $q, q, p$ ]

True

Sequent is Axiom!

No contradiction.

Close branch.

52. yields:

LHS: [ $q$ ]

RHS: [ $q, q, p$ ]

51. yields:

LHS: [ $q, \sim p$ ]

RHS: [ $q, q$ ]

50. yields:

LHS: [ $\sim q, q, \sim p$ ]

RHS: [ $q$ ]

right split:

Performing iteration with:

LHS:  $\sim q, q, q$

RHS:  $q$

Non-atomic proposition found on LHS...

Iterating on the left-hand side...

54. Left-  $\sim$  on  $\sim q$  with

LHS:  $q, q$

RHS:  $q$

Performing iteration with:

LHS:  $q, q$

RHS:  $q$

LHS & RHS contain only atoms!

Checking if an atom in LHS also resides in RHS:

[q,q]

[q,q]

Checking if q is contained in

[q,q]

True

Sequent is Axiom!

No contradiction.

Close branch.

54. yields:

LHS: [q,q]

RHS: [q,q]

50. yields:

LHS: [ $\sim$ q,q,q]

RHS: [q]

43. yields:

LHS: [ $(\sim$ p  $\vee$  q), $\sim$ q,q]

RHS: [q]

42. yields:

LHS: [(p  $\vee$  q), $(\sim$ p  $\vee$  q), $\sim$ q]

RHS: [q]

31. yields:

LHS: [ $\sim$ q,(p  $\vee$  q), $(\sim$ p  $\vee$  q), $\sim$ q]

RHS: []

30. yields:

LHS: [(p  $\vee$   $\sim$ q), $\sim$ q,(p  $\vee$  q), $(\sim$ p  $\vee$  q)]

RHS: []

3. yields:

LHS: [((p  $\vee$  q)  $\wedge$   $(\sim$ p  $\vee$  q)),(p  $\vee$   $\sim$ q), $\sim$ q]

RHS: []

2. yields:

LHS: [ $(\sim$ p  $\vee$   $\sim$ q),((p  $\vee$  q)  $\wedge$   $(\sim$ p  $\vee$  q)),(p  $\vee$   $\sim$ q)]

RHS: []

1. yields:

LHS: [(((p  $\vee$  q)  $\wedge$   $(\sim$ p  $\vee$  q))  $\wedge$  (p  $\vee$   $\sim$ q)), $(\sim$ p  $\vee$   $\sim$ q)]

RHS: []

0. yields:

LHS: [(((p  $\vee$  q)  $\wedge$   $(\sim$ p  $\vee$  q))  $\wedge$  (p  $\vee$   $\sim$ q))]

RHS: [ $\sim$ ( $\sim$ p  $\vee$   $\sim$ q)]