

Definition 1 (Gamma-function) $\Gamma(n)$ is an extension of the factorial function to complex and real arguments. This relation is given by $\Gamma(n) = (n - 1)!$ for all $n \in \mathbb{N}$.

Definition 2 (Integral form) For $z \in \mathbb{R}$, $z > 0$, the Γ -function is given by:

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$$

Definition 3 (Probability density function (pdf)) A function $f(x)$ is a pdf if and only if:

$$P(a \leq X \leq b) = \int_a^b f(x) dx, f(x) \geq 0, \forall x \in \mathbb{R}$$

Result 1 $f(x)$ is a probability density function if and only if the following are true:

$$f : \mathbb{R} \rightarrow [0, 1], f(x) \geq 0 \tag{1}$$

$$\int_{-\infty}^{\infty} f(x) dx = 1 \tag{2}$$

Notation 1 Z denotes the standard normal distribution (a normal distribution with mean 0 and standard deviation 1). The pdf of this distribution is given by:

$$f_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Definition 4 ($\Gamma(\alpha, \beta)$ -distribution) The $\Gamma(\alpha, \beta)$ -family is given by the following pdf:

$$f_{\Gamma(\alpha, \beta)}(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$$